



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



- 1 Two uniform smooth spheres A and B of equal radii have masses $4m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides directly with sphere B . After the collision, the momentum of A is three times the momentum of B .

Find the value of the coefficient of restitution e .

[4]

[illegible]



- 2 A particle P of mass m is moving in a horizontal circle with angular speed ω_1 on the smooth inner surface of a hemispherical shell of radius r . The angle between the upward vertical and the normal reaction of the surface on P is θ_1 , where $\tan \theta_1 = \frac{3}{4}$.

When the angular speed is increased to ω_2 , the angle between the upward vertical and the normal reaction of the surface on P becomes θ_2 , where $\tan \theta_2 = \frac{4}{3}$.

Find the ratio $\frac{\omega_1}{\omega_2}$.

[4]

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- 3** A particle P is moving in a straight horizontal line. At time t s, the displacement of P from a fixed point O on the line is x m and the velocity of P is v ms⁻¹. The acceleration of P is $\frac{1}{2}(v^2 + 4)$ ms⁻² in the direction PO . Initially P is at O and is moving with velocity 2 ms⁻¹.

(a) Find an expression for x in terms of t .

[5]

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(b) Find the time when P next goes through O .

[2]



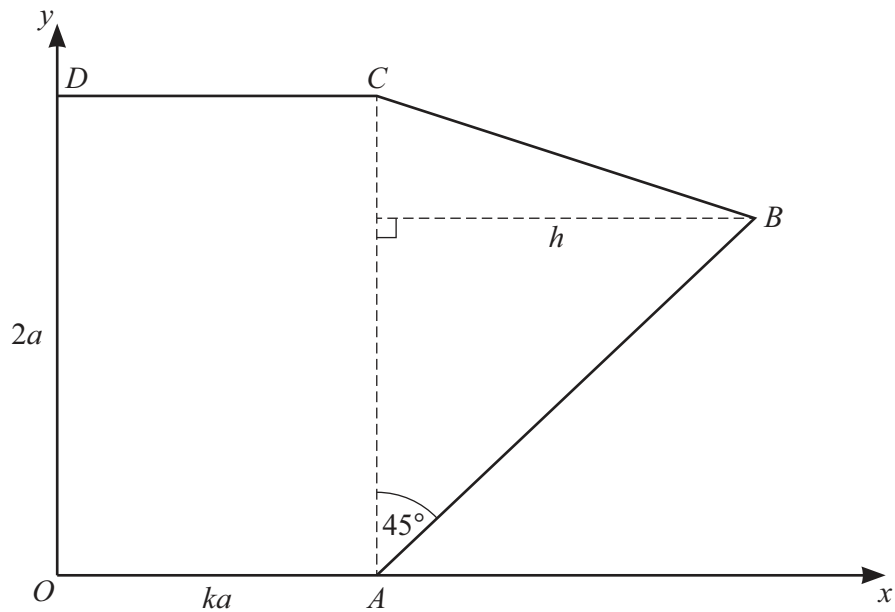


- 4 One end of a light elastic string of natural length a and modulus of elasticity $5mg$ is attached to a fixed point O . Two particles, P and Q , of masses m and $4m$ respectively are attached to the other end of the string and they hang vertically in equilibrium. Particle Q is then detached from the string, hence releasing particle P from rest.

Find, in terms of a , the length of the string when the speed of particle P is first equal to $\sqrt{\frac{7}{5}ag}$. [6]

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[illegible]



A uniform lamina $OABCD$ consists of a rectangle $OACD$ and a triangle ABC . The length of OA is ka , the length of OD is $2a$, the height of triangle ABC is h and angle CAB is 45° (see diagram). Relative to axes through O , parallel and perpendicular to OA as shown, the centre of mass of triangle ABC is $(\bar{x}, \bar{y})$.

- (a) Show that $\bar{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\bar{y}$. [3]

[illegible]



The lamina $OABCD$ is placed vertically on its edge OA on a horizontal plane.

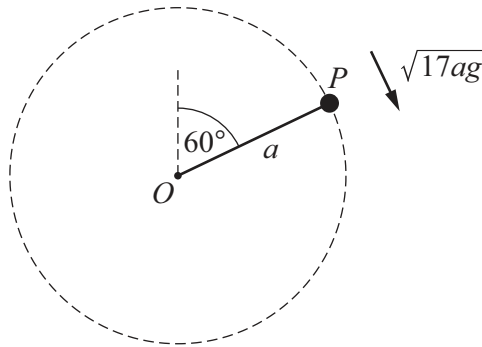
- (b) Find, in terms of a and k , the set of values of h for which the lamina is in equilibrium. [4]

[illegible]

It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

- (c) Find, in terms of a , the coordinates of the centre of mass of the triangle ABC . [2]

[illegible]



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . Initially P is held with the string taut and making an angle of 60° with the upward vertical through O . The particle P is projected perpendicular to the string in a downwards direction with speed $\sqrt{17ag}$. It then starts to move along a circular path in a vertical plane with centre O (see diagram). At the lowest point of its path, vertically below O , the particle P collides with a stationary particle Q .

- (a) Find, in terms of a and g , an expression for the speed of P immediately before the collision with Q . [2]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

As a result of the collision, P rebounds and moves back along a circular path with centre O . The string becomes slack when P reaches the point on the circle vertically above O .

- (b) Find, in terms of a and g , an expression for the speed of P immediately after the collision with Q . [3]

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The mass of particle Q is km and the collision between P and Q is perfectly elastic.

- (c) Find the value of k . [3]

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- 7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is 25ms^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A , the direction of motion of P makes an angle of 45° with the downward vertical through A .

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A . [5]

[illegible]

At point A , the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of 45° to the horizontal.

- (b) Find the speed of P immediately before it collides with the barrier. [3]

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- (c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. [4]

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[illegible]





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